in particular, through the action of laser radiation on the gas [9].

NOTATION

k, Boltzmann's constant; T_g, gas temperature; m_i, n_i, Q_i, mass of a molecule, density of molecules in the condensate, and heat of evaporation of a molecule, respectively, of component i; n_{gi}, density of molecules of component i in the gaseous phase; L, r, length and radius, respectively, of the cylindrical channel; $l = L/r$; D_k, Knudsen coefficient of diffusion of component i; x, dimensionless coordinate.

LITERATURE CITED

1. V. E. Minaichev, Cryogenic Vacuum Pumps [in Russian], Énergiya, Moscow (1976).
2. Ya. I. Frenkel', The Kinetic Theory of Liquids, Peter Smith.
3. K. P. Shumskii, A. I. Myalkin, and I. S. Maksimovskaya, Principles of the Design of Vacuum Sublimation Apparatus [in Russian], Mashinostroenie, Moscow (1967).
4. M. N. Kogan and N. K. Makashev, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 6 (1971).
5. P. Clausing, Ann. Phys., 12, 961 (1932).
6. A. I. Ivanovskii, Tr. Tsentr. Aerol. Obs., No. 56 (1964).
7. A. V. Luikov, T. L. Perel'man (Perelman), V. V. Levdanskii (Levdansky), V. G. Keitsina, and N. V. Pavlyukevich, Int. J. Heat Mass Transfer, 17, No. 9 (1974).
8. V. V. Levdanskii, in: Problems of the Theory of Transfer Processes [in Russian], Inst. Teplo- i Massoobmena, Minsk (1977).
9. N. V. Karlov and A. M. Prokhorov, Usp. Fiz. Nauk, 123, No. 1 (1977).

INTERNAL RELAXATION IN PHASE INTERACTION IN A DISPERSED SYSTEM

E. M. Tolmachev

UDC 536.70:529.5

Corrections have been made to the relaxation times for heat and mass transfer between phases in dispersed systems, which incorporate the thermal conduction and diffusion within the particles.

The relaxation formalism in the thermodynamics of irreversible processes has been developed most fully by Meixner [1] and leads to expressions of the form

$$\chi^{\omega} = \chi^{\infty} + \frac{\chi^{0} - \chi^{\infty}}{1 + i\omega\tau}, \tag{1}$$

which allow one to calculate the generalized susceptibility χ^{ω} for a thermodynamic system in relation to the frequency of variation of the parameters and the values of the susceptibility in two limiting states: equilibrium χ^0 and frozen-in χ^{∞} together with the relaxation times for the corresponding process.

Recently, the relaxation approximation has been fairly widely used to describe the behavior of dispersed systems subject to external perturbations [2-5]. One then obtains formally expressions of the type of (1) with the following assumptions: a) the dispersed system is a single-phase medium with internal degrees of freedom that describe the exchange between phases; b) each of the phases is in internal equilibrium, even if there is no equilibrium between the phases; and c) it is assumed in calculating the relaxation time that the thermal or diffusion Biot number is negligibly small, and that this is caused not by smallness in the heat or mass transfer coefficients but by large values of the thermal conductivity or diffusion coefficients for the particles. Clearly, assumptions b and c are related.

In fact, the thermal conduction or thermal diffusion within the particles may be the rate-limiting processes, particularly for relatively large particles. In the relaxation approximation, this does not alter the general form of (1) but influences the relaxation time

---

Kirov Urals Polytechnical Institute, Sverdlovsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 37, No. 4, pp. 609-612, October, 1979. Original article submitted November 21, 1978.

0022-0841/79/3704-1150$07.50

τ. This effect can be incorporated correctly by considering precisely the propagation of heat in a particle, whose shape is taken as spherical and whose surface temperature varies harmonically. The diffusion problem can then be considered by simple replacement of the thermal characteristics by diffusion ones, by virtue of the identity of the differential equations and boundary conditions.

We therefore have the following thermal-conduction equation for spherical symmetry:

$$\frac{\partial\theta_1}{\partial t} = a_1 \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial\theta_1}{\partial r}\right) \tag{2}$$

with the boundary condition

$$\theta_1(R,\ t) = \delta T_{10} \exp(i\omega t). \tag{3}$$

The solution to (2) that satisfies the boundary condition is

$$\theta_1(r,\ t) = \delta T_{10} \frac{X}{x} \cdot \frac{\operatorname{sh} x}{\operatorname{sh} X} \exp(i\omega t), \tag{4}$$

where

$$x = r\sqrt{\frac{i\omega}{a_1}};\quad X = R\sqrt{\frac{i\omega}{a_1}}. \tag{5}$$

The amount of heat absorbed by a particle in unit time is given by the Fourier law

$$q(t) = -4\pi R^2 \lambda_1 \left.\frac{\partial\theta_1}{\partial r}\right|_{r=R} \tag{6}$$

or on the basis of (4)

$$q(t) = 4\pi R^3 \lambda_1 \delta T_{10} \frac{i\omega}{a_1} \cdot \frac{L(X)}{X} \exp(i\omega t), \tag{7}$$

where

$$L(X) = \operatorname{cth} X - \frac{1}{X} \tag{8}$$

is a Langevin function [6].

The temperature distribution of (4) describes the temperature wave propagating into a particle and decaying in accordance with sinh x/x; we can, as is usual [7], introduce the penetration depth $\delta(\omega)$ of the temperature wave and assume only that a spherical layer of thickness $\delta(\omega)$ participates in the heat transfer to the surrounding phase. It is more convenient however, to consider the particle as a whole as having a bulk temperature equal to the surface temperature and incorporate the nonuniform temperature distribution via an effective specific heat $(\rho_1 c_1)^\omega$ via the expression

$$q(t) = \frac{4}{3}\pi R^3 (\rho_1 c_1)^\omega \left.\frac{\partial\theta_1}{\partial t}\right|_{r=R} = \frac{4}{3}\pi R^3 (\rho_1 c_1)^\omega i\omega \delta T_{10} \exp(i\omega t), \tag{9}$$

where (3) has been used.

Comparison of (7) and (9) gives

$$(\rho_1 c_1)^\omega = \rho_1 c_1 \frac{3}{X} L(X). \tag{10}$$

The following are the limiting relations for the effective specific heat:

$$(\rho_1 c_1)^0 = \lim_{\omega\to 0} (\rho_1 c_1)^\omega = \rho_1 c_1;\quad (\rho_1 c_1)^\infty = \lim_{\omega\to\infty} (\rho_1 c_1)^\omega = 0. \tag{11}$$

The temperature-relaxation time τ_q is found usually from the heat balance equation for a particle with Newtonian heat transfer at the surface:

$$\frac{4}{3}\pi R^3 (\rho_1 c_1)^\omega \left.\frac{\partial\theta_1}{\partial t}\right|_{r=R} = -4\pi R^2 \alpha \left(\theta_1|_{r=R} - \theta_0\right). \tag{12}$$

We then get the effective temperature-relaxation time as

$$\tau_q^\omega = \frac{(\rho_1 c_1)^\omega R}{3\alpha} = \tau_q^0 \frac{3}{X} L(X), \tag{13}$$

where τ_q^0 is the equilibrium temperature-relaxation time from the viewpoint of internal equilibrium. Then the generalized susceptibility for the dispersed medium takes the form

$$\chi^\omega = \chi^\infty + \frac{\chi^0 - \chi^\infty}{1 + i\omega\tau^\omega}. \tag{14}$$

In a case of temperature relaxation, the dispersion term $(1 + i\omega\tau^\omega)^{-1}$ is written on the basis of (13) as

$$\frac{1}{1 + i\omega\tau_q^\omega} = \frac{1}{1 + i\omega\tau_q^0 \frac{3}{X} L(X)}. \tag{15}$$

We use the expansion of the Langevin function [6] to get for small values of ω on the basis of (5) that

$$\frac{1}{1 + i\omega\tau_q^\omega} = \frac{1}{1 + i\omega\tau_q^0 (1 - i\omega\tau_\lambda)} \approx \frac{1}{1 + i\omega \dfrac{\tau_q^0}{1 + i\omega\tau_\lambda}}, \tag{16}$$

where

$$\tau_\lambda = R^2/15a_1 \tag{17}$$

may be called the internal temperature-relaxation time.

The Langevin function differs little from unity at high frequencies, so from (13) we have

$$\frac{1}{1 + i\omega\tau_q^\omega} = \frac{1}{1 + \sqrt{i\omega}\, \dfrac{3\tau_q^0 \sqrt{a_1}}{R}} = \frac{1}{1 + \sqrt{i\omega}\, \psi_1}, \tag{18}$$

where

$$\psi_1 = \frac{\sqrt{\lambda_1 \rho_1 c_1}}{\alpha}. \tag{19}$$

Note that $(\sqrt{\omega}\psi_1)^{-1}$ is the modified Biot number for a stationary-periodic state [7].

Therefore, the dispersion law of (16) differs little from the ordinary law of (1) for low frequencies if the thermal conductivity of the particle material is not too low, whereas there is a substantial change at high frequencies.

NOTATION

χ, generalized susceptibility; ω, angular frequency; τ, relaxation time; t, time; T_{10}, mean surface temperature of a particle; δT_{10}, amplitude of surface temperature variation; θ, relative temperature; r, radial coordinate; R, particle radius; a, thermal diffusivity; ρ_1, particle density; c, specific heat; λ, thermal conductivity; q, heat absorbed by a particle per unit time; α, heat-transfer coefficient. Superscripts: ω, dynamic quantity; 0, equilibrium state ($\omega \to 0$); ∞, for $\omega \to \infty$. Subscripts: 0, motion phase; 1, particles, q, heat transfer; λ, heat conduction.

LITERATURE CITED

1. J. Meixner, Kolloid.-Z., 134, 3 (1953).
2. I. S. Radovskii, Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1970).
3. E. M. Tolmachev, Candidate's Dissertation, S. M. Kirov Ural Polytechnic Institute, Sverdlovsk (1972).
4. G. P. Yasnikov and V. S. Belousov, Inzh.-Fiz. Zh., 34, No. 5 (1978).
5. G. P. Yasnikov and V. S. Belousov, Inzh.-Fiz. Zh., 34, No. 6 (1978).
6. A. I. Ansel'm, Principles of Statistical Physics and Thermodynamics [in Russian], Nauka, Moscow (1973).
7. A. V. Lykov, Heat and Mass Transfer (Handbook) [in Russian], Énergiya, Moscow (1972).